

Spiral Review

Determine which quadrants the angles could be in.

1.) $\sin\theta > 0$

I, II

2.) $\tan\theta < 0$

II, IV

3.) $\csc\theta < 0$

III, IV

4.) $\sec\theta > 0$

I, IV

p.284 4.4 Trigonometric Functions of Any Angle

Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2}$.
* r is always (+) *

$$\sin\theta = \frac{y}{r}$$

$$\csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r}$$

$$\sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

Student will be able to find the exact values of the six trigonometric functions.

Example 1: The point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions.

$$\begin{matrix} x & y \\ (8, & 15) \end{matrix}$$

① Find r .

$$r = \sqrt{8^2 + 15^2}$$

$$r = \sqrt{289}$$

$$r = 17$$

② Find six trig functions.

$$\sin \theta = \frac{15}{17}$$

$$\cos \theta = \frac{8}{17}$$

$$\tan \theta = \frac{15}{8}$$

$$\csc \theta = \frac{17}{15}$$

$$\sec \theta = \frac{17}{8}$$

$$\cot \theta = \frac{8}{15}$$

Student will be able to determine the quadrant the angle lies in.

Example 2: State the quadrant in which θ lies.

a.) $\sec \theta < 0$ and $\cot \theta < 0$

II, III

II, IV

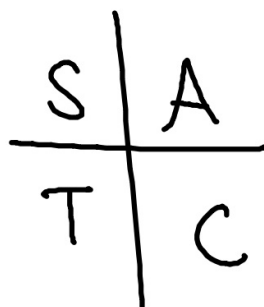
II

b.) $\tan \theta > 0$ and $\csc \theta < 0$

I, III

III, IV

III



Student will be able to use the fundamental identities of trigonometry.

Example 3: Use the given value and the trigonometric identities to find the remaining trigonometric functions of the angle.

a.) $\cos\theta = -\frac{3}{7}, \sin\theta < 0$

① Quadrant?
III (-, -)

② Use the Pythagorean Identity with the given #.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{3}{7}\right)^2 = 1$$

$$\sin^2\theta + \frac{9}{49} = 1$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{40}{49}}$$

$$\sin\theta = -\frac{2\sqrt{10}}{7}$$

③ What 4 functions do you know?

$$\sin\theta = -\frac{2\sqrt{10}}{7} \quad \csc\theta = \frac{7}{-2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$\csc\theta = \frac{7\sqrt{10}}{-20}$$

$$\cos\theta = -\frac{3}{7} \quad \sec\theta = -\frac{7}{3}$$

④ what do we still need?

$$\tan\theta = \frac{-2\sqrt{10}}{-\frac{3}{7}} \quad \cot\theta = \frac{3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$$

$$\tan\theta = \frac{2\sqrt{10}}{3} \quad \cot\theta = \frac{3\sqrt{10}}{20}$$

Student will be able to use the fundamental identities of trigonometry.

b.) $\sec\theta = -\frac{4}{3}, \cot\theta > 0$

QIII (-, -)

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \tan^2\theta = \left(-\frac{4}{3}\right)^2$$

$$1 + \tan^2\theta = \frac{16}{9}$$

$$\sqrt{\tan^2\theta} = \sqrt{\frac{7}{9}}$$

$$\tan\theta = \frac{\sqrt{7}}{3}$$

$$\tan\theta = \frac{\sqrt{7}}{3} \quad \cot\theta = \frac{3}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

$$\cot\theta = \frac{3\sqrt{7}}{7}$$

$$\cos\theta = -\frac{3}{4} \quad \sec\theta = -\frac{4}{3}$$

$$\sec\theta = -\frac{4}{3}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(-\frac{3}{4}\right)^2 = 1$$

$$\sin^2\theta + \frac{9}{16} = 1$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{7}{16}}$$

$$\sin\theta = -\frac{\sqrt{7}}{4}$$

$$\csc\theta = \frac{4}{-\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$\csc\theta = -\frac{4\sqrt{7}}{7}$$

Student will be able to find two solutions of each equation.

Example 4: Find two solutions of each equation. Give your solutions in both degrees and radians. Do not use a calculator!

a.) $\cos\theta = \frac{\sqrt{2}}{2}$ **I, IV**

$$\frac{\pi}{4}, \frac{7\pi}{4}$$

$$45^\circ, 315^\circ$$

b.) $\tan\theta = 1$ **I, III**

$$\frac{\pi}{4}, \frac{5\pi}{4}$$

$$45^\circ, 225^\circ$$

c.) $\cot\theta = -\sqrt{3}$ **II, IV**

$$\frac{5\pi}{6}, \frac{11\pi}{6}$$

$$150^\circ, 330^\circ$$

d.) $\csc\theta = 2$ **I, II**

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

$$30^\circ, 150^\circ$$

Turn-in:

p. 289 (18, 88, 106)

HW:

p. 289 (15-25, 85-89, 103-107 odds)