

Examples:


FACTORIZING QUADRATIC EQUATIONS IN FORM $x^2 + bx + ?$

Develop a flair for completing the square.

Ex. $x^2 - 12x + ?$

Step 1. Find $\frac{1}{2}$ of the coefficient of x . -6	Step 2. Square the result of Step 1. $(-6)^2 = 36$
Step 3. Add the result of Step 2 to $x^2 + bx$. $x^2 - 12x + 36$	Step 4. Now factor the perfect square. $(x - 6)^2$

THERE IS NOTHING SO FAIR AS THE PERFECT SQUARE!



$$\begin{aligned}
 \text{a.) } f(x) &= (2x^2 + 4x) - 7 \\
 &= (2x^2 + 4x) - 7 \\
 &= 2(x^2 + 2x + 1) - 7 - 2 \\
 &= 2(x+1)^2 - 9
 \end{aligned}$$

$\frac{2}{2} = 1^*$
 $(1)^2 = 1$

$$\begin{aligned}
 \text{b.) } g(x) &= (-7x^2 + 14x) - 20 \\
 &= (-7x^2 + 14x) - 20 \\
 &= -7(x^2 - 2x + 1) - 20 + 7 \\
 &= -7(x-1)^2 - 13
 \end{aligned}$$

$-\frac{2}{2} = -1^*$
 $(-1)^2 = 1$

Conic Sections

Parabola: $(x-h)^2 = 4p(y-k)$
 $(y-k)^2 = 4p(x-h)$

Circle: $(x-h)^2 + (y-k)^2 = r^2$

Ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Example 1:

$$y = x^2 - 8x + 12 \quad (\text{parabola})$$

$$\begin{array}{ccc} & -12 & \\ & & -12 \end{array}$$

$$y - 12 \overset{+16}{=} x^2 - 8x + 16$$

$$\frac{-8}{2} = -4x$$

$$(-4)^2 = 16$$

$$\boxed{y + 4 = (x - 4)^2}$$

Example 2:

$$x = 2y^2 + 8y - 12 \quad (\text{parabola})$$

$$\begin{array}{ccc} & +12 & \\ & & +12 \end{array}$$

$$x + 12 = 2y^2 + 8y$$

$$x + 12 \overset{+8}{=} 2(y^2 + 4y + 4)$$

$$\frac{4}{2} = 2x$$

$$(2)^2 = 4$$

$$x + 20 = 2 \frac{(y + 2)^2}{2}$$

$$\boxed{\frac{1}{2}(x + 20) = (y + 2)^2}$$

Example 3:

$$8y - 6x + x^2 = -y^2 - 24$$
$$+y^2 \quad +y^2$$

$$y^2 + 8y + x^2 - 6x = -24 \quad (\text{circle})$$

$$y^2 + 8y + \underline{16} + x^2 - 6x + \underline{9} = -24 + \underline{16} + \underline{9}$$

$$\boxed{(y+4)^2 + (x-3)^2 = 1}$$

Example 4:

$$121x^2 + 4y^2 - 242x - 80y + 37 = 0 \quad (\text{ellipse})$$

$$121x^2 - 242x \quad + 4y^2 - 80y \quad = -37$$

$$121(x^2 - 2x + \underline{1}) + 4(y^2 - 20y + \underline{100}) = -37 + \underline{121} + \underline{400}$$

$$\frac{121(x-1)^2}{484} + \frac{4(y-10)^2}{484} = \frac{484}{484}$$

$$\boxed{\frac{(x-1)^2}{4} + \frac{(y-10)^2}{121} = 1}$$

Example 5:

$$9x^2 - y^2 - 36x - 6y + 18 = 0 \quad (\text{hyperbola})$$

$$9x^2 - 36x - y^2 - 6y = -18$$

$$9(x^2 - 4x + 4) - (y^2 + 6y + 9) = -18 + 36 - 9$$

$$\frac{9(x-2)^2}{9} - \frac{(y+3)^2}{9} = \frac{9}{9}$$

$$\boxed{\frac{(x-2)^2}{1} - \frac{(y+3)^2}{9} = 1}$$